



## Control performance of separately excited DC motors

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### General Note



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### ABSTRACT

There are various control methodologies for direct current motors with each method possessing its attendant peculiarity. In this work, the control strategies, transfer functions, and performance analysis of open loop control, closed loop speed control and inner current loop controlled separately excited DC motor. For better stability, a closed loop speed controller is used. The chosen speed control technique is the Proportional-Integral controller. The open-loop response shows that the machine is easily affected by external disturbances along with accompanying transients. The best control strategy is using inner-current loop control which gives more stability and better response. The analysis is presented using MATLAB/SIMULINK.

**Keywords:** MATLAB/Simulink, DC motors, open and closed loops, PID controller

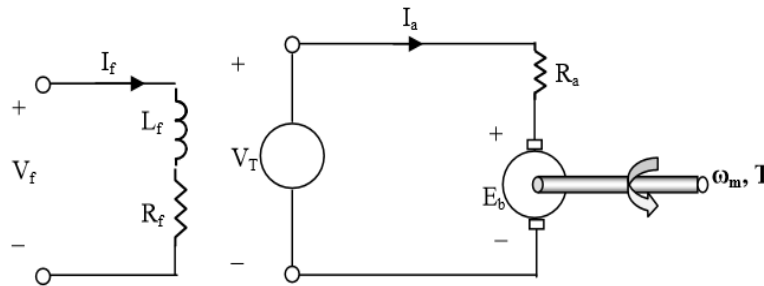
## 1. INTRODUCTION

DC motors can be found in practically all industrial and domestic applications such as in the production and processing of paper pulp, textile industries, electric vehicles, propulsion and in public transport such as TRAM (trolley) and METRO. This is because they exhibit a wide speed range, good speed regulation, starting and accelerating torques in excess of 400% or the rated value, less complexity and its usually inexpensive drives. Some works [1-6] have looked at different aspects of the operations of the machine. While studies in [1-3] touched on the performances following the responses arising from the software employed, the papers in [4-6] focused mainly on the control schemes. Conventionally, most separately excited DC motors are controlled by variation of the armature voltage by different methods such as armature resistance control, in which an external resistance is connected to the armature windings. Another more efficient method of control of DC motors is by the use of DC drives. These drives alter the output parameter (usually the voltage) depending on a control signal applied to them. DC drives commonly referred to as converters have improved the controllability, efficiency, current carrying capabilities of DC motors. The increasing complexity of industrial processes demands greater flexibility from electrical machines in terms of special characteristics and speed control [7]. In order to avoid discrepancies that arise when simulating speed control of DC motors in Simulink using static converters and the DC motor block, the DC motors and the controller are modeled mathematically and the control performance is evaluated in open loop, closed loop, inner current loop and the results are presented using MATLAB/SIMULINK.

### Mathematical Model of A Separately Excited DC Motor (SEDCM)

In order to accurately model the SEDCM, the following assumptions are made: the rotor and shaft are assumed to be rigid, the input is the armature voltage  $V$  in Volts (driven by a voltage source), the friction torque is proportional to shaft angular speed, the magnetic field is constant and thus the torque is directly proportional to the armature current, the demagnetizing effect of the armature reaction is assumed negligible and the measured variables are the angular velocity of the shaft  $\omega$  in radians per second, and the shaft angle  $\theta$  in radians.

### System Equations



**Figure 1** Separately excited DC motor

For a separately excited DC motor with constant field flux whose equivalent circuit is shown in figure 1, the armature voltage can be obtained using Kirchhoff's voltage law on the armature winding (ignoring brush drop) as

$$V_a = R_a i_a + L_a \frac{di_a}{dt} + E_b \quad (1)$$

But

$$E_b = K_b \omega_m \quad (2)$$

Where  $E_b$  = emf induced in the armature windings,  $i_a$  = armature current,  $R_a$  = armature winding resistance,  $L_a$  = armature winding inductance,  $K_a$  = motor constant and  $\omega_m$  = speed of motor. Similarly, applying KVL on the field winding gives,

$$V_f = R_f i_a + L_f \frac{di_f}{dt} \quad (3)$$

where,  $V_f$  = Field winding excitation voltage,  $L_f$  = inductance of field winding,  $R_f$  = resistance of field winding and  $i_f$  = field current. From Newton's second law, the mechanical rotation of the motor can be modeled using the torque equation below;

$$T_m = T_L + B_m \omega_m + J_m \frac{d\omega_m}{dt} \quad (4)$$

$$\text{But } T_m = K_b i_a \quad (5)$$

where,  $T_m$ =motor torque,  $T_L$ = external load torque,  $J_m$ =Motor inertia,  $B_m$ =Coefficient of friction and  $K_b$  is the motor torque constant dependent on the number of poles, effective number of conductors.

### Transfer function of a SEDCM

By applying the Laplace transform, equations 1 and 4 can be written as:

$$V_a(s) = R_a I_a(s) + S L_a i_a(s) + E_a(s) \quad (6)$$

$$T_m(s) = T_l(s) + B_m \Omega_m(s) + S J_m \Omega_m(s) \quad (7)$$

From equation (6), we have,

$$K_m \approx K_b \approx K_a \quad (8)$$

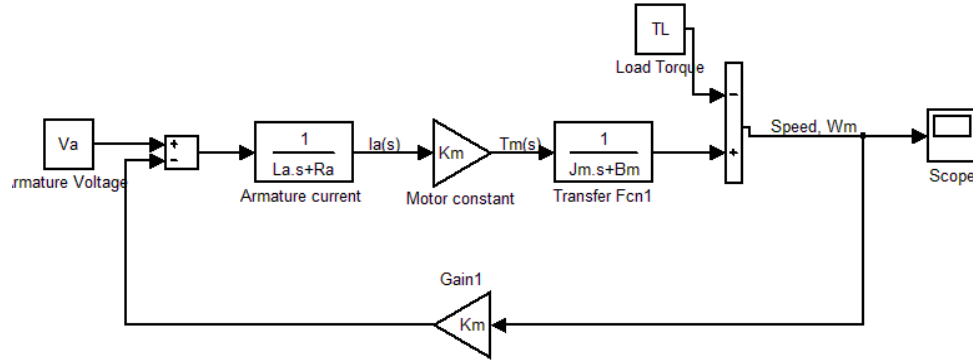
$$I_a(s) = \frac{V_a(s) - K_m \Omega_m(s)}{S L_a(s) + R_a} \quad (9)$$

where  $S$  denotes the Laplace operator. Making  $\Omega_m(s)$  the subject of equation 7 gives

$$\Omega_m(s) = \frac{T_m(s) - T_l(s)}{S J_m(s) + B_m} \quad (10)$$

The corresponding gain of the system with speed as the output and armature voltage as input is:

$$G = \frac{\omega_m(s)}{V_a(s)} = \frac{K_m}{(S L_a(s) + R_a)(S J_m(s) + B_m) + K_m^2} \quad (11)$$



**Figure 2** SEDCM transfer function model in Simulink

The equations of the SEDCM already stated in equations 1 to 5, can be written as

$$\frac{di_a}{dt} = \frac{1}{L_a} (V_a - R_a i_a - E_b) \quad (12)$$

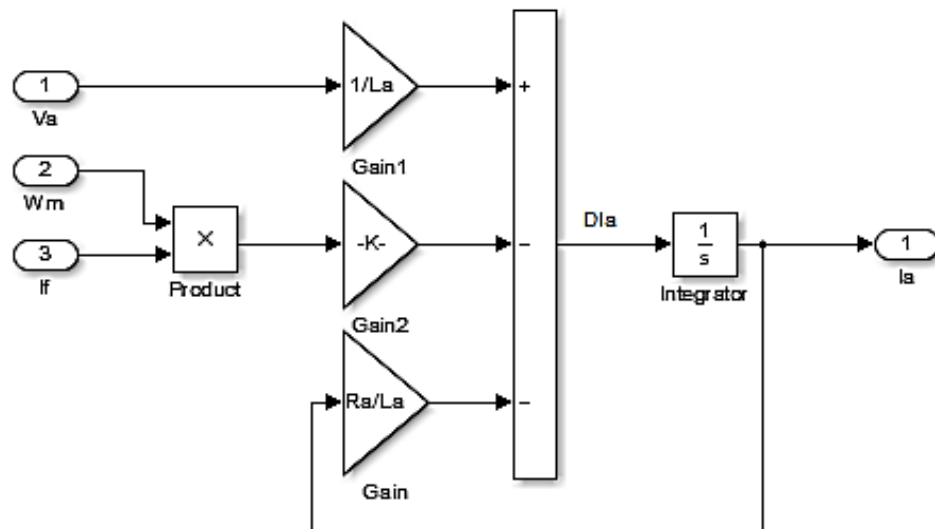
Since  $E_b = K_b \omega_m$ , equation (10) can further be re-expressed as,

$$\frac{di_a}{dt} = \frac{1}{L_a} (V_a - R_a i_a - K_b \omega_m) \quad (13)$$

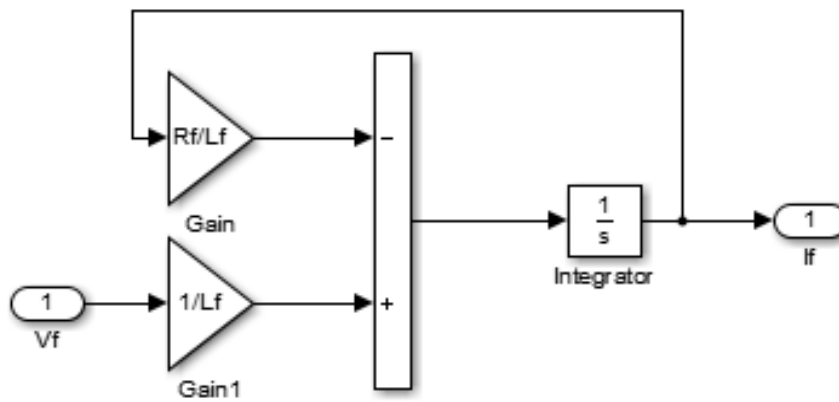
$$\frac{di_f}{dt} = \frac{1}{L_f} (V_f + R_f i_f) \quad (14)$$

$$\frac{d\omega_m}{dt} = \frac{1}{J_m} (T_m - T_l + B_m \omega_m) \quad (15)$$

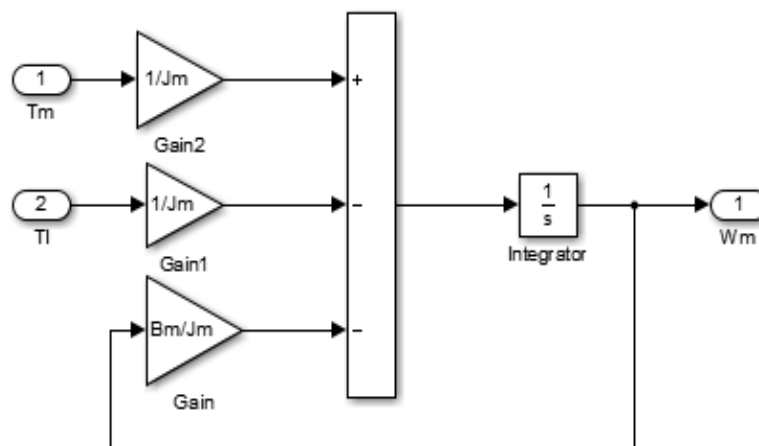
From the above equations, Simulink sub-models of the separately excited DC motor are shown in following figures 2 -6.



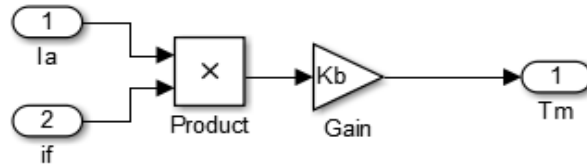
**Figure 3** Armature current Subsystem



**Figure 4** Field current subsystem



**Figure 5** Speed subsystem



**Figure 6** Motor torque subsystem

## 2. ANALYSIS OF THE CONTROL PATTERNS

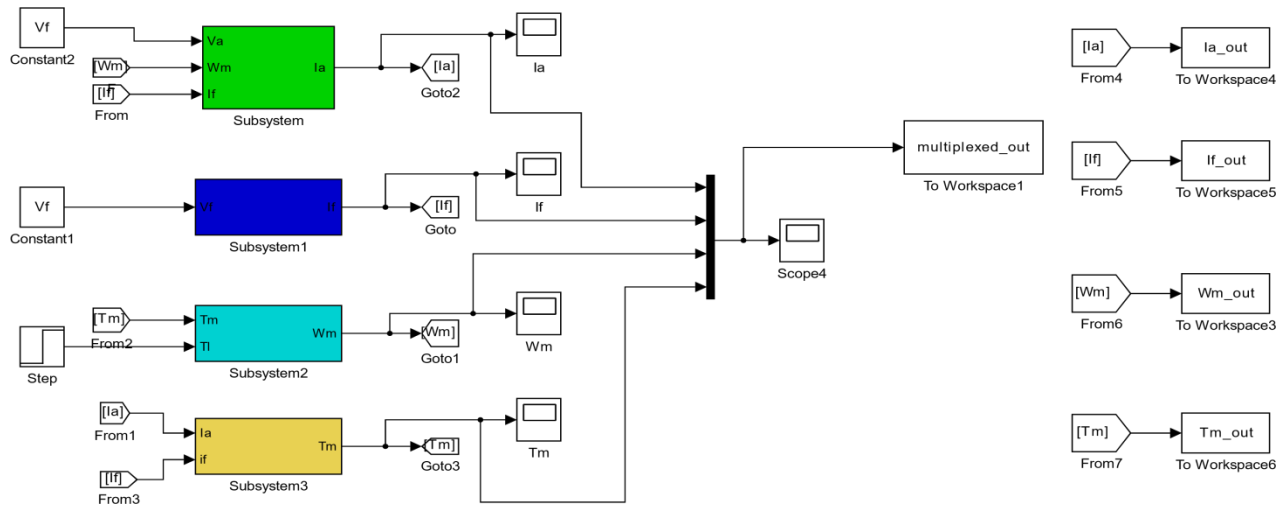
### Open Loop Control1

Under open loop conditions, the motor is easily affected by external disturbances such as applied load torque. In order to return the motor to steady state speed, an operator needs to re-adjust the armature voltage so that the motor speed returns to the steady state speed. The separately excited DC motor operates in an unstable manner. This can be seen in the results.

### Closed Loop Control

For better control of the SEDCM, a closed loop control methodology is adopted with a suitable speed compensator. In this work, only the Proportional and the Proportional-Integral controller will be discussed. P-I controller has the optimum control dynamics including zero steady state error, fast response (short rise time), no oscillations and higher stability (figure 7).

The fundamental concept of frequency domain analysis is the transfer function which expresses the relationship between the system output in Laplace transform  $Y(s)$  to the system input  $U(s)$ .



**Figure 7** Simulink Model of the SEDCM using the system equations

The gain of a second order system is represented by the following transfer function

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b_0}{a_2 s^2 + a_1 s + a_0} \quad (16)$$

$$G(s) = \frac{(\omega_n)^2}{s^2 + 2\xi\omega_n s + (\omega_n)^2} \quad (17)$$

Comparing this with the gain above, we have

$$G(s) = \frac{\frac{b_0}{a_0}}{s^2 + \frac{a_1}{a_2}s + \frac{a_0}{a_2}} \quad (18)$$

Therefore,

$$\frac{a_1}{a_2} = 2\zeta\omega_n \quad (19)$$

$$\frac{a_0}{a_2} = \omega_n^2 \quad (20)$$

$$\omega_n = \sqrt{\frac{a_0}{a_2}} \quad (21)$$

The roots of the characteristic equation can be obtained from the quadratic formula as

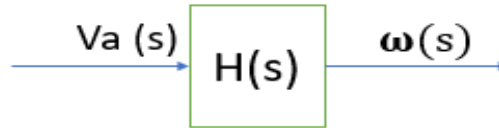
$$s_{1,2} = \zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1} \quad (22)$$

From the above, in order to know the nature or shape of the system we must vary  $\zeta$ . If the real part of the equation (22) is negative, the system is stable; otherwise the system is unstable.

In tuning the PI controller, the settling time for DC motor is expected to be 2 seconds. The settling time of a second order system is given by

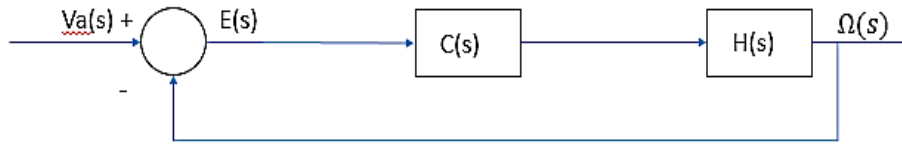
$$t_s = \frac{4}{\xi\omega_n} \quad (23)$$

From the transfer function model in Figure 2, the block diagram of the system can be simplified (fig 8 – 12) thus:



**Figure 8** Simplified block diagram representation

Introducing the proportional controller in the forward path, we have



**Figure 9** Introducing the gain of the controller

$C$  = Gain of the proportional controller which is given by

$$C = \frac{K_p s + K_i}{s} \quad (24)$$

Since  $L_a$  is very small compared to the other parameters,  $L_a \sim 0$ . this simplifies the order of the transfer function to

$$H(s) = \frac{K_m}{R_a J_m s + (R_a B_m + K_m^2)} \quad (25)$$

Expressing the gain in the form below,

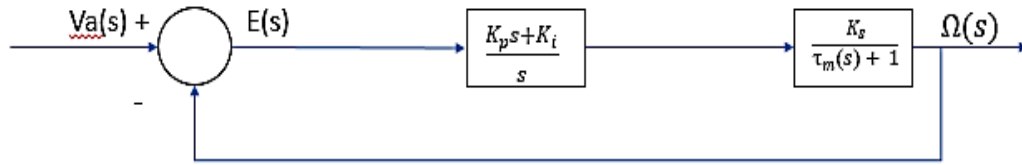
$$G(s) = \frac{\Omega(s)}{V_a(s)} = \frac{K_s}{\tau_m(s) + 1} \quad (26)$$

where,

$$K_s = \frac{(R_a + K_m)}{(R_a B_m + (K_m)^2)} \quad (27)$$

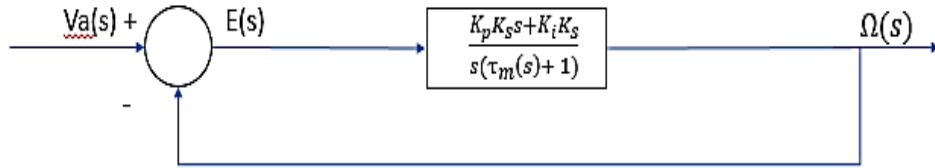
$$\tau_m = \frac{(R_a J_m)}{(R_a B_m + (K_m)^2)} \quad (28)$$

If the speed-voltage transducer has an efficiency of  $K_f$ , the feedback gain becomes  $K_f$ . In this case it has been calculated to be 0.138.



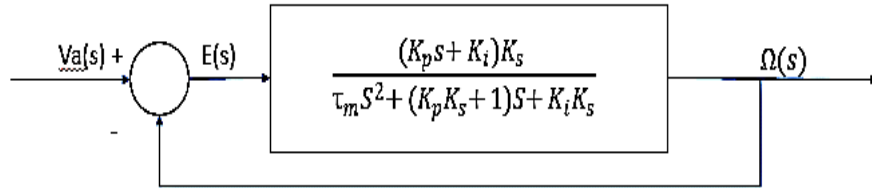
**Figure 10** Simplified block diagram

This can be reduced to the figure below



**Figure 11** Reduced block diagram of figure 10.

This can further be reduced using block diagram reduction techniques to



**Figure 12** Transfer function of the DC motor with PI controller

Modifying the characteristic equation and comparing with the canonical representation of a second order system, we have

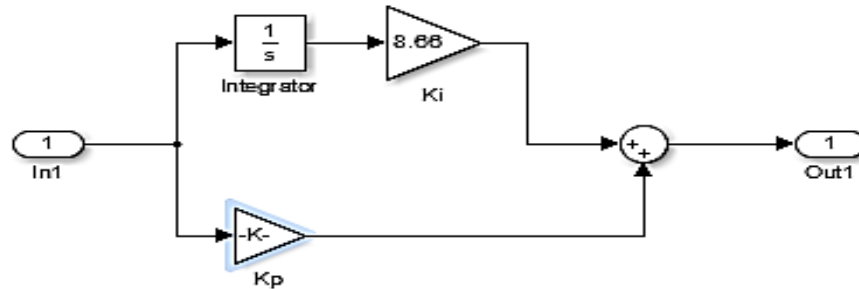
$$\frac{V_a}{\Omega} = \frac{(K_p s + K_i) K_s (\tau_m)^{-1}}{s^2 + (\tau_m)^{-1} (K_p K_s + 1) s + K_i K_s (\tau_m)^{-1}} \quad (29)$$

so that,

$$(\omega_n)^2 = \frac{K_i K_s}{\tau_m} \quad (30)$$

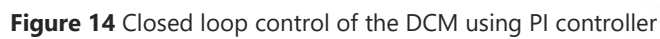
$$2\xi\omega_n = \frac{K_s K_p + 1}{\tau_m} \quad (31)$$

The PI controller for the system has been modeled and shown below in figure 13.

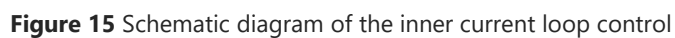


**Figure 13** Modeled PI controller

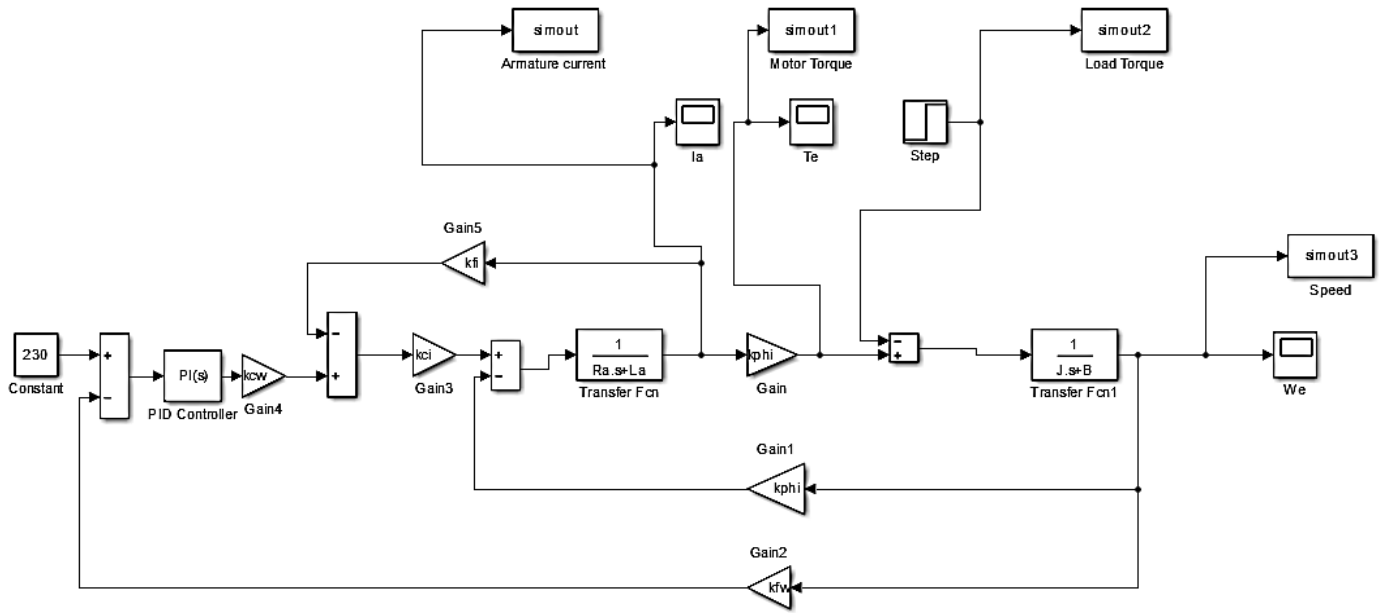
The closed loop control is shown below with the PI controller set to the values of  $K_i$  and  $K_p$  obtained above (fig. 14).



Improvement in speed control can be obtained with Inner Current Loop Control method, whereby armature current is fed back to the input [8]. A closed loop speed control scheme with inner current control is shown in the figure 15 & 16.







**Figure 16** inner current loop controls

**Table 1** The parameters for the DC motor are given below

Machine Parameters										
Variable	$R_a$	$L_a$	$R_f$	$J_m$	$B_m$	$T_L$	$K_b$	$V_f$	$V_a$	$L_f$
Value	$2\ \Omega$	16.2mH	$180\ \Omega$	0.117 kg.m <sup>2</sup>	0.001 Nms <sup>2</sup>	10 Nm	0.699 Nm/A <sup>2</sup>	12V	24V	71.47H

### 3. RESULTS

#### Pi Controller

For an optimal condition for most systems, according to Root-Locus analysis, the value of  $\xi$  is between 0.5-0.7. In this work, the settling time  $t_s$  is expected to be 2 seconds and  $\xi$  is taken to be 0.6. From equations 23, 27 and 28 we have

$$\omega_n = \frac{4}{\xi t_s} = \frac{4}{0.6 \times 2} = 3.33 \quad (33)$$

$$K_s = \frac{0.699}{(2.581 \times 0.002953) + 0.699^2} = 1.4086 \quad (34)$$

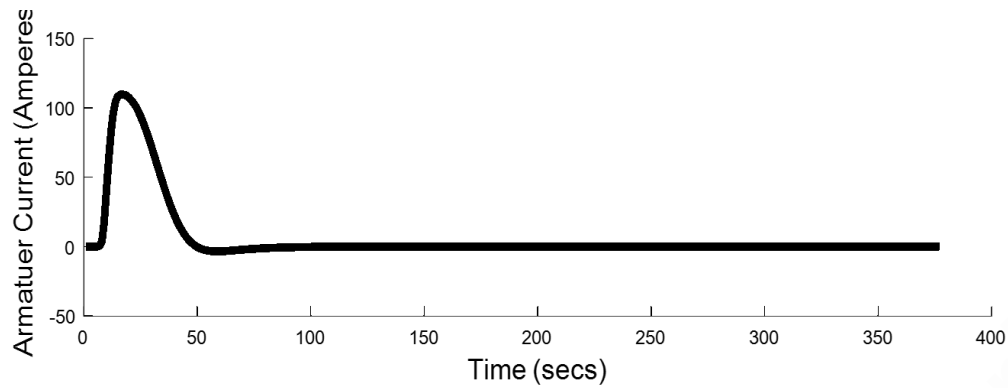
$$\tau_m = \frac{2.581 \times 0.022125}{(2.581 \times 0.02953) + 0.699^2} = 0.1011 \quad (35)$$

$$K_i = \frac{3.33^2 \times 1.1011}{1.4086} = 8.66 \quad (36)$$

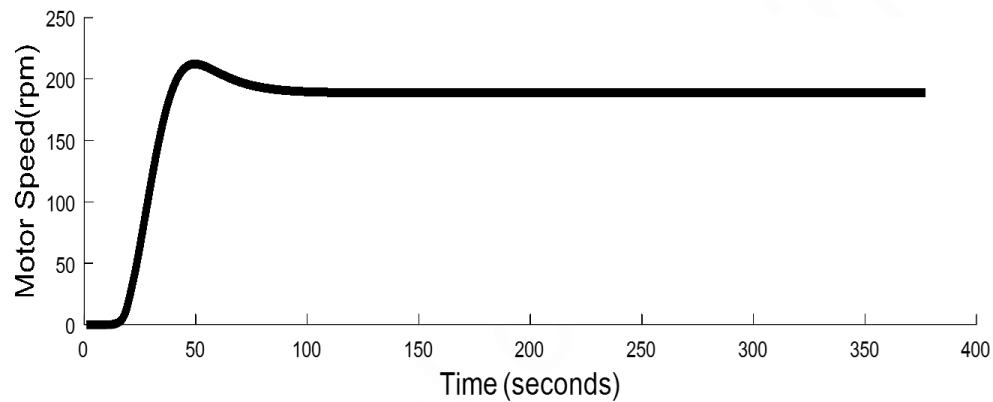
$$K_p = \frac{(2 \times 0.6 \times 3.33 \times 1.011) - 1}{8.66} = 0.35103 \quad (37)$$

#### Open Loop Response

Under no load conditions the following results are obtained (fig. 17 – 18).



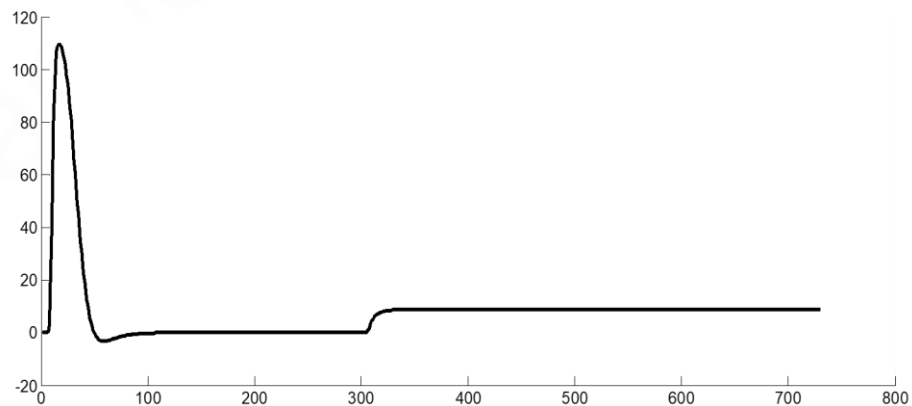
**Figure 17** Armature current response



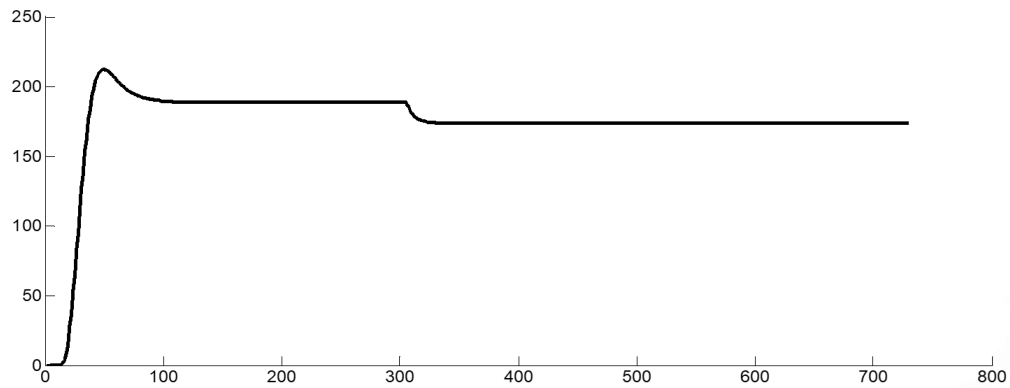
**Figure 18** Speed response

### On-Load Performance

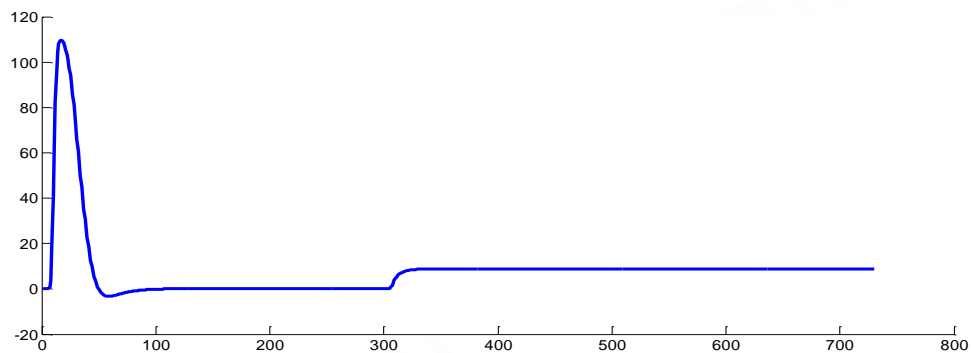
When a load of 10Nm is applied to the motor the speed of the motor changes and the current builds up to accommodate the change in load while the speed drops (fig. 19 – 21).



**Figure 19** Armature current response



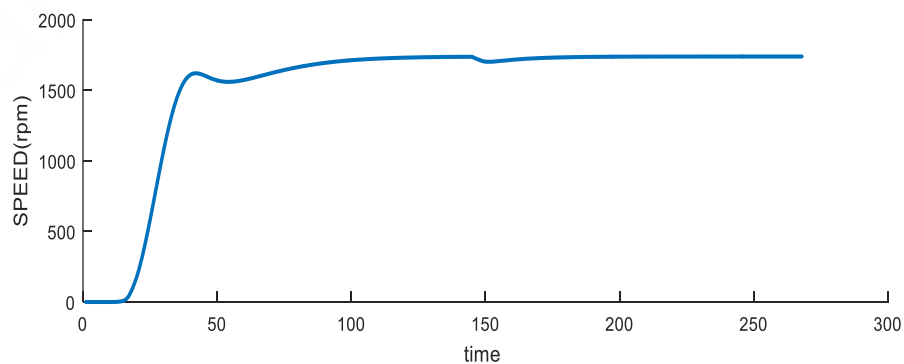
**Figure 20** Speed response



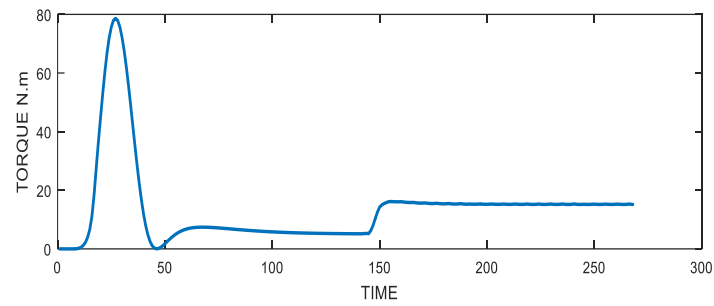
**Figure 21** Torque response

### Closed Loop Response

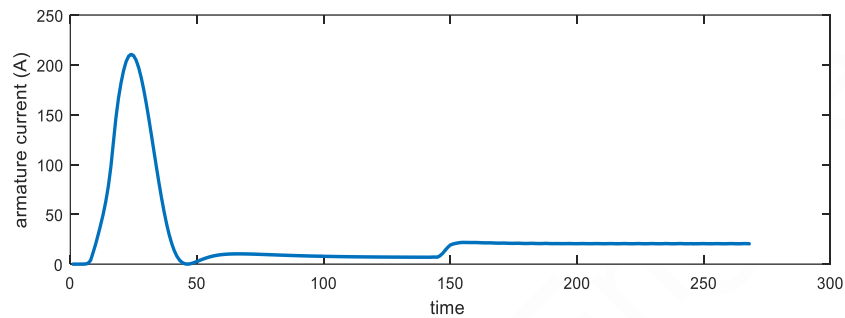
The motor was simulated with the load torque applied after 5 seconds. It is seen from the response of the speed shown below that on application of load after 5 seconds the speed of the motor experiences a short dip but returns to its steady state change. This is shown below (figure 22 – 24).



**Figure 22** Speed response

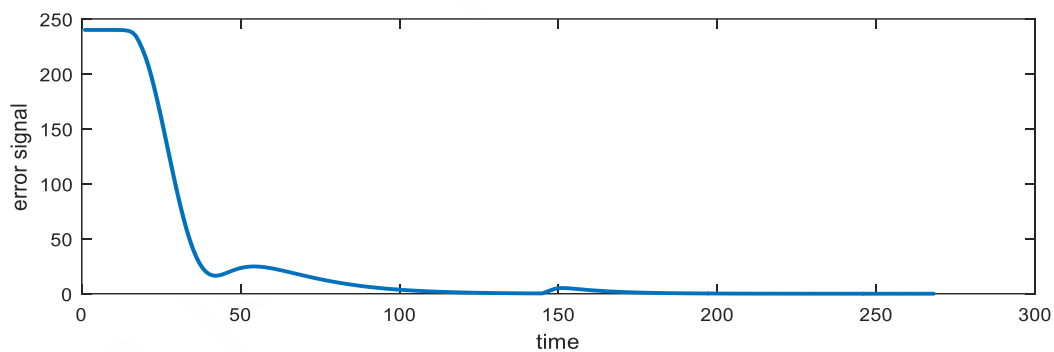


**Figure 23** Torque response



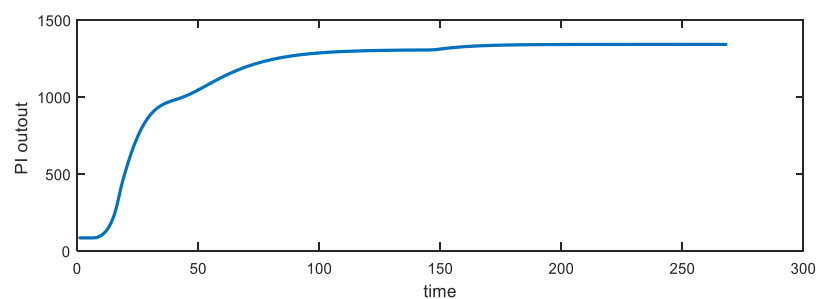
**Figure 24** Armature current response

The motor is able to achieve this stability as a result of the PI controller that compensates for the error signal generated when the speed of the motor tends to change as a result of applied load (after 5s). The error signal is shown below (fig. 25).



**Figure 25** Error signal in as seen by the PI controller

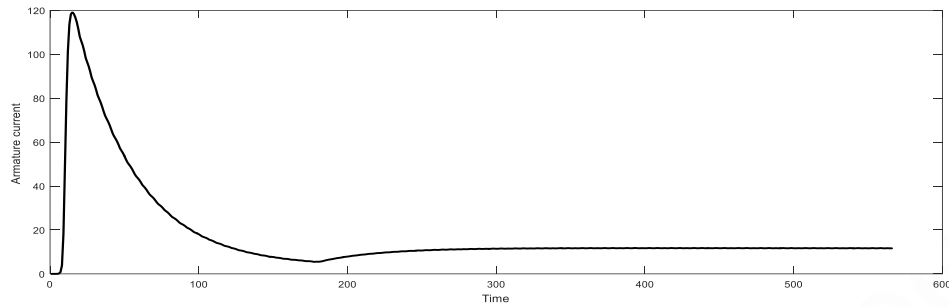
This error is compensated and the input to the motor increased to allow for an increased speed in order to return the motor to steady state speed. The output of the PI controller is shown below (fig. 26).



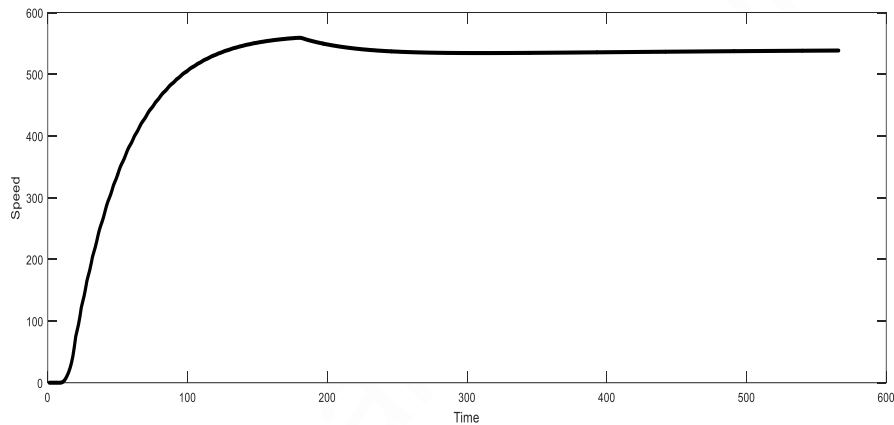
**Figure 26** output of the PI controller

### Inner Current Loop Control

Using the following values for the inner current loop,  $k_{\phi i}=1.3369$ ;  $k_{cw}=733.57$ ;  $k_{fw}=0.4244$ ;  $k_{ci}=555.98$ ;  $k_{fi}=0.33$ ;  $K_p=0.00023961268604131$   $k_i=4.42179513226605e-05$ , a more stable response is obtained as shown below (fig. 27 & 28).



**Figure 27** Armature current response



**Figure 28** Speed response

From the response shown above, it can be seen that the DC motor gives a fairly stable response even when a stepped input load of 10Nm is applied at time  $t=3$ secs.

### Stability, controllability and observability analysis

The stability of the motor can be analyzed using the Routh-Hurwitz stability criterion (table 2). If the system is stable all the elements in the column will have the same sign [9]. The approximate transfer of the DC motor under study is given below

$$G = \frac{0.699}{1.895s^2 + 0.2502s + 0.4906} \quad (38)$$

From the characteristic equation of the transfer function

$$1.895s^2 + 0.2502s + 0.4906 = 0 \quad (39)$$

**Table 2** Routh-Hurwitz Table

$S^2$	1.895	0.4906	0
$S^1$	0.2502	0	0

$S^0$	0.1227	0	0
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From the above it can be seen that the elements in the first column are positive and of the same sign, therefore the system is stable. As the system turns out stable, different types of controller can be used with the machine to check performance. By definition, a system is said to be controllable at some initial time for any initial state  $X(0)$ , if there exists some input  $U(t)$  that can drive the system to any final state in a desired set time  $X_d[10]$ . In order to evaluate the controllability of the machine, the actual state space representation of the machine can be obtained using the inbuilt MATLAB command:

[E, F, J, H]=tf2ss (num1, den1); ctrb (E,F), obsv (E,F.'). From the MATLAB code, the motor is seen to be controllable and observable.

#### 4. CONCLUSION

There are various control methodologies for Direct current control motors with each method possessing its pros and cons. The choice of which depends on the design specifications and use. Two classical methods of control of DC motors have been discussed and simulated in this work. It can be seen from the simulation results obtained that the use of controllers such as PI controller affects the stability and response of DC machines. From the simulation results, it can be seen that the performance characteristics of the SEDCM is greatly improved when the inner current loop control method is adopted. This method offers the best control benefits of increased speed response (reduced settling time) and improved (reduced) Speed Regulation. DC-DC converters are the most commonly used devices for speed control of DC motors since they directly affect the armature voltages which on the other hand affect the speed of the DC motor.

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**Conflicts of Interest:** The authors declare no conflict of interest.

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